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Critical dynamics of self-organizing Eulerian walkers

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The model of self-organizing Eulerian walkers is numerically investigated on the square lattice. The critical exponents for the distribution of a number of steps (τ_l) and visited sites (τ_s) characterizing the process of transformation from one recurrent configuration to another are calculated using the finite-size scaling analysis. Two different kinds of dynamical rules are considered. The results of simulations show that both versions of the model belong to the same class of universality with the critical exponents $\tau_l = \tau_s = 1.75 \pm 0.1$. [S1063-651X(97)09201-5]

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To understand the nature and physical origins of the self-organized criticality (SOC) [1], a number of different models have been suggested in the past few years. Typically, these models evolve according to the prescribed dynamical rules into the SOC state, where they show spatiotemporal self-similarity and are characterized by long-range correlations. In the critical state, the systems pass from one stable configuration to another through the avalanches that play an essential role in the organization of the dynamical criticality.

In this paper, we numerically investigate critical dynamics of a cellular automaton model recently proposed by Priezzhev *et al.* [2]. This model has some common features with the well-known Abelian sandpile model (ASM) [3,4], but differs in rules that govern the motion of particles on the lattice.

The model of the self-organizing Eulerian walkers on the square lattice is defined as follows. We associate with each site i of the two-dimensional $L \times L$ lattice an arrow directed up, right, down, or left with respect to i. We start with an arbitrary initial configuration of arrows on the lattice. Initially, we drop a particle on the randomly chosen site i. The succeeding steps the particle performs are determined by the following rules: (i) the particle coming to a site j turns the arrow clockwise by the right angle, (ii) then makes a step along the new direction of the arrow to the neighbor site, and (iii) if the new direction points out the lattice, the particle leaves the system. These rules are applied until the particle eventually leaves the lattice. Then, we go on by adding a new particle and so on. In this type of dynamics, the movement of the particle affects the medium and in turn is affected by the medium.

On the lattice with closed boundary conditions, the particle never leaves the system and finally gets into a limit cycle in which it passes each bond in both directions only once. Walks of this type are known as Euler circuits [5].

Let us consider the lattice with open boundary conditions. The set of bonds marked by arrows form a graph G. Adding the particles followed by their movement through the lattice changes the configuration of arrows and organizes the system into the SOC state [2]. This critical state is a collection of recurrent configurations of a Markov process because the motion of a succeeding particle depends only on the final configuration of arrows produced by its predecessor. Each element of this recurrent set can be obtained from the previous one by adding a particle followed by its movement and leaving the lattice. It turns out that for any recurrent configuration, the graph G is a spanning tree on the lattice. Thus the set of recurrent configurations is in a one-to-one correspondence with spanning trees [2].

At each intermediate step, the moving particle can destroy a spanning tree and form a loop of arrows. At this moment the system leaves the recurrent set. Eventually, after a finite number of steps the particle reconstructs the structure of a spanning tree. The interval between the destruction and restoration of the spanning tree can be called an *avalanche of cyclicity*. During the walk of the particle, the system passes several times from one recurrent configuration to another through the avalanches of cyclicity. This process is similar to the avalanche dynamics of sand in the ASM, where avalanches also reconstruct the recurrent configurations, which can also be represented by spanning trees [4].

At the beginning of an avalanche of cyclicity the last turned arrow closes a loop. Then, the trajectory of the particle covers the interior of the loop. During this walk each inner arrow turns four times, whereas the arrows forming the loop turn in such a way that the direction of the loop gets reversed. It might occur that a moving particle, after escaping from one closed loop, may form a new loop. Due to this structure of the walk, the number of steps in an avalanche is equal to $k(4n+1)$, where k is the number of loops constituting the avalanche and $n = 0,1, \ldots$. It is possible to prove that the avalanche may consist of only one or two loops. This fact explains the line doubling in the distribution of the steps in the avalanche of cyclicity (Fig. 1).

To investigate the avalanche process in the models of self-organizing Eulerian walkers, we studied them numerically with high statistics. For each distribution of avalanches we considered up to 30×10^6 events on the square lattices of linear size L from 120 to 400. Simulations always started from the regular initial configuration in which all arrows were directed up.

In Fig. 2 we present the double logarithmic plot of the distribution $P(s)$ of the number of visited sites in the ava-

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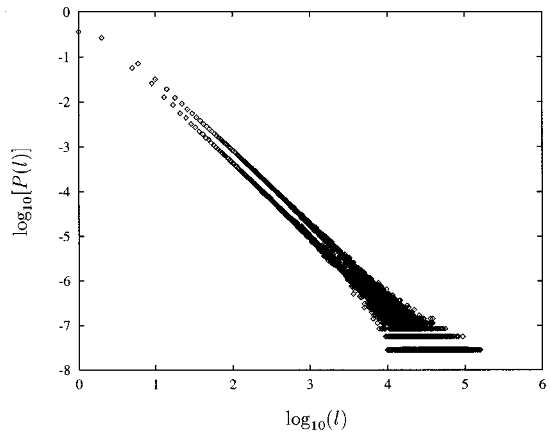

FIG. 1. Distribution $P(l)$ of the number of steps in avalanches on the square lattice of linear size $L=400$.

lanche for the lattice size $L=400$. The analysis of the data shows that this distribution obeys the power law

$$P(s)\sim s^{-\tau_s}. \quad (1)$$

To estimate the critical exponents, we have performed a finite-size scaling analysis [6,7], assuming that the distribution functions scale with the lattice size L as

$$P(x,L)=L^{-\beta}f(x/L^{\nu}), \quad (2)$$

where $f(y)$ is a universal scaling function and β and ν are critical exponents that describe the scaling of the distribution function.

To reduce the fluctuations of the data, we integrated each distribution over exponentially increasing bin lengths. For the integrated bin distribution we have [8]

$$D(s)=\int P(x)dx\sim s^{-(\tau_s-1)}. \quad (3)$$

Plotting $D(s,L)L^{\beta_s}$ versus $sL^{-\nu_s}$ on a double logarithmic scale, as is shown in Fig. 3 for the different lattice sizes L, we obtained that the best data collapse corresponds to $\beta_s=1.5\pm0.05$, $\nu_s=2.0\pm0.05$ (Fig. 4). The scaling relation

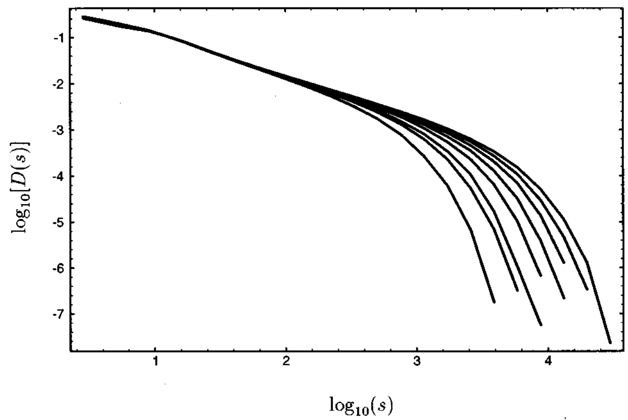

FIG. 3. Integrated distributions $D(s)$ for the eight lattice sizes with L ranging from 120 to 400.

for the critical exponents $\tau_s=\beta_s/\nu_s+1$ gives the value $\tau_s=1.75\pm0.05$.

In the same way, we investigated the distribution $P(l)$ of the number of steps performed by the particle in the avalanche for the different lattice sizes L. There is an explicit power-law behavior in these distributions (Fig. 1)

$$P(l)\sim l^{-\tau_l}, \quad (4)$$

with a finite-size cutoff. We applied the finite-size scaling analysis to the integrated distributions and obtained $\tau_l=1.7\pm0.05$ from the best data collapse.

We also investigated a slightly modified model. The difference from the previous one is in the order of turns of the arrow. In the case when the turns form the sequence up-down-left-right-up, we found a similar power law for avalanche distributions.

To find the critical exponents of this power law from finite-size scaling analysis, we integrated again these distributions over exponentially increasing bin lengths. The critical exponents $\tau_l=1.72\pm0.05$ and $\tau_s=1.8\pm0.05$ have been obtained from the best data collapse for the distribution of steps and visited sites, respectively.

In conclusion, we numerically investigated the model of self-organizing Eulerian walkers on the square lattice. The

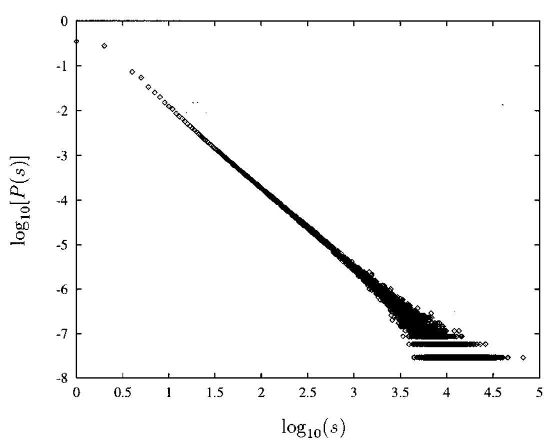

FIG. 2. Distribution $P(s)$ of the number of visited sites in avalanches on the square lattice of linear size $L=400$.

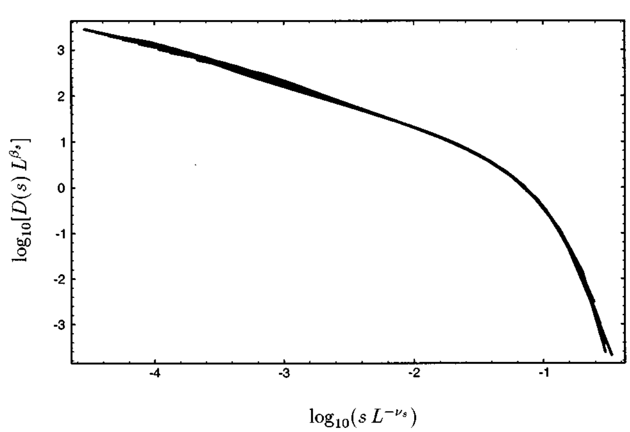

FIG. 4. Finite-size scaling for the integrated distributions $D(s)$.

dynamics of the model organizes the medium of the system and builds up spatiotemporal complexity. We obtained explicit power-law distributions in two slightly different versions of the model. We calculated the critical exponents for the distribution of a number of visited sites (τ_s) and number of steps (τ_l) in avalanches of cyclicity. These exponents are equal within a small uncertainty. We argue that the critical exponents for these models within small errors belong to the same class of universality and have a surprisingly large value, 1.75 ± 0.1, in comparison to the known exponent for the ASM ($\tau=5/4$) [9].

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